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New high-resolution digital sinusoidal oscillator structure with extremely low frequency and sensitivity

AHMAD A. HIASAT† and ABEDULAH M. AL-KHATEEB†

An efficient extremely low frequency digital sinusoidal oscillator with high resolution and extremely low sensitivity is presented. The proposed oscillator structure can generate frequencies much lower than all structures known in the literature without the need to increase the width of the multiplier coefficient. The proposed oscillator utilizes two low sensitivity complex digital oscillators with two different multiplier coefficients. The simulation results verify the analytical results and demonstrate the performance as measured in terms of total harmonic distortion.

1. Introduction

Tunable digital sinusoidal oscillators are widely used in many applications. Their applications include areas such as communications, music synthesis, radar, control, and system identification, to mention a few. The digital oscillators exhibit the advantages of digital techniques, which are stability, flexibility and low cost. Moreover, the continuous development of new algorithms makes it desirable to have digital sinusoidal oscillators.

The conventional digital sinusoidal oscillator utilizes one look-up table in which the samples of a complete cycle of a sine wave are stored in a read-only memory (ROM) and read at appropriate time intervals (National Semiconductor Corp. 1981, Garcia 1986). The frequency resolution of the ROM methods was improved in Schanerberger and Awad (1990), where the effective table length was increased. A high frequency resolution digital sinusoidal oscillator was presented in Al-Ibrahim and Al-Khateeb (1998), using look-up tables.

Digital sinusoidal oscillators can also be generated in a recursive fashion without the need for ROM. A typical digital sinusoidal oscillator is a second-order recursive digital filter with poles on the unit circle in the complex Z-plane. Stable sinusoidal signals based on a second-order difference equation were investigated in Furuno *et al.* (1975) and Mitra *et al.* (1975). The quantization of the multiplier coefficient and the arithmetic operations results are due to the finite word-length constraints that appear in the practical implementation of digital oscillator structure. Consequently, frequency and amplitude distortions are always present at the output waveform. A second-order recursive digital structure with stable oscillation was analysed in Hartimo (1983). A digital oscillator structure with very low roundoff errors was presented in Abu-El-Haija and Al-Ibrahim (1986 a, b). An efficient low-frequency digital sinusoidal oscillator was presented in Al-Ibrahim and Al-Khateeb (1996 a). A digital sinusoidal oscillator with low and uniform frequency spacing was also presented in Al-Ibrahim and Al-Khateeb (1997).

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A digital oscillator structure with extremely low frequency, extremely low sensitivity and high frequency resolution is proposed in this paper. The proposed oscillator evaluates the frequency difference between two extremely low sensitivity complex digital oscillators Al-Ibrahim and Al-Khateeb (1996 b) with two different multiplier coefficients. The frequency resolution of a digital oscillator is a function of the width of the multiplier coefficient. Therefore, to increase the frequency resolution, the width of the multiplier coefficient must be increased.

In the proposed oscillator structure, the width of the multiplier coefficients is the same as that of the complex digital oscillator. It is shown that the proposed structure is very efficient in generating extremely low frequencies. Moreover, the structure exhibits the advantage of extremely low sensitivity to coefficient quantization. The total harmonic distortion (THD) of the proposed structure is of the same order as that of the extremely low sensitivity complex oscillator, with low harmonic distortion.

2. Extremely low sensitivity digital sinusoidal oscillator

The extremely low sensitivity digital oscillator is a complex oscillator that has two digital integrators and a multiplier which are arranged in a closed-loop fashion (Al-Ibrahim and Al-Khateeb 1996 b). Each integrator has a transfer function given by

$$H_I(z) = \frac{G}{1 - z^{-1}} \tag{1}$$

where G is the gain of the integrator. The integrator can be implemented as an accumulator. The gain G can be implemented simply, by ignoring the least significant b_i bits and considering the remaining most significant bits of the accumulator for the next stage. Therefore, the output of the integrator is equal to the content of the accumulator multiplied by the gain G. The accumulator saves its contents for the next iteration. Consequently, the least significant bits of the accumulator are not actually dropped. The multiplier coefficient of the complex oscillator determines the frequency of the generated sinusoid and is implemented as

$$\alpha = 2\cos\theta = 2\left(1 - \frac{K2^{-b_3} G_1 G_2}{2}\right) \tag{2}$$

where G_1 and G_2 are the gains of the integrators, $K2^{-b_3}$ is the multiplier coefficient $(0 \le K2^{-b_3} \le 1)$, and b_3 is the number of bits used in implementing the multiplier (multiplier width). The gains of the integrators G_1 and G_2 can be implemented as simple shifts (i.e. $G_i = 2^{-b_i}$, $i = 1, 2$). The multiplier coefficient is given by

$$2\cos\theta = 2\left(1 - \frac{K2^{-(b_1+b_2+b_3)}}{2}\right) \tag{3}$$

The ideal version of the complex oscillator is shown in the block diagram of figure 1. Each integrator can be considered as an accumulator with a gain of the form $G_i = 2^{-b_i}$, $i = 1, 2$. The multipliers required in the implementation of the complex oscillators are $K2^{-b_3}$ and $K2^{-(b_2+b_3)}/\sin\theta$. Using such gains for the integrators and multipliers is equivalent to distributing the bits over the whole structure to increase the frequency resolution and simplify the implementation. By solving the difference

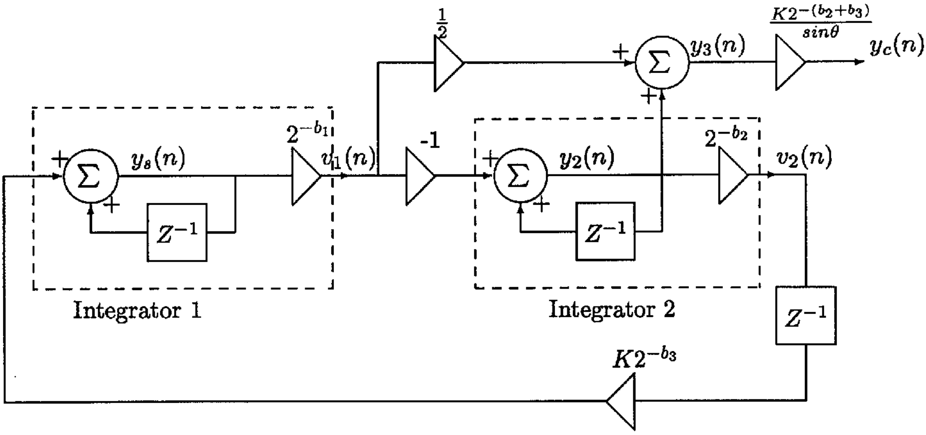

Figure 1. Block diagram of the complex digital oscillator.

equations governing the operation of the sine–cosine loop under the assumption that all initial conditions are zeros except $y_2(-1)$, it can be shown (Al-Ibrahim and Al-Khateeb 1996 b) in the absence of quantization error that $y_c(n)$ and $y_s(n)$ are the real and imaginary parts of the complex exponential $K2^{-(b_2+b_3)}/\sin\theta$ and are given by

$$y_s(n) = \frac{K2^{-(b_2+b_3)}}{\sin\theta} y_2(-1)\sin(n\theta), \quad n \geq 0 \tag{4}$$

$$y_c(n) = \frac{K2^{-(b_2+b_3)}}{\sin\theta} y_2(-1)\cos(n\theta), \quad n \geq 0 \tag{5}$$

where

$$\theta = \cos^{-1}\left(1 - \frac{K2^{-(b_1+b_2+b_3)}}{2}\right) \tag{6}$$

It is obvious that only two multipliers are required to implement the complex oscillator. The integrators can be implemented without the need for a multiplier. This structure is better than the complex digital oscillator (multiple-output direct-form digital oscillator) of Fliege and Wintermantel (1992), since three multipliers are required to implement their complex oscillator.

3. Analysis of the complex digital oscillator

Generally speaking, the oscillators actually constructed suffer from two main problems, namely, coefficient quantization effects and roundoff errors (Abu-El-Haija and Al-Ibrahim 1986 b). These problems are due to finite wordlength constraints.

3.1. *Sensitivity*

The sensitivity is the effect of an error in implementing the multiplier coefficient on the frequency of the generated sine and cosine signals. This is due only to the finite set of values that is possible for the multiplier coefficient. The error in frequency $\Delta\theta$ of the generated complex wave is expressed as a function of the error in representing the multiplier coefficient ΔK due to finite wordlength constraints. The error $\Delta\theta$ was approximated by Al-Ibrahim and Al-Khateeb (1996 b):

$$\Delta\theta = -\frac{2^{-(b_1+b_2+b_3)}}{2\sin\theta}\Delta K \tag{7}$$

The sensitivity to coefficient quantization of the complex digital oscillator was shown to be lower by a factor of $2^{-(b_2+b_3)}$ than that of the direct form digital oscillator (Fliege and Wintermantel 1992).

3.2. *Roundoff errors*

The finite wordlength constraints cause roundoff errors to be present as a sequence to suit the allowed number of bits. This quantization error will accumulate with time, causing the oscillator output to deviate from the ideal output in both amplitude and frequency. Figure 2 shows a linearized model of the complex digital oscillator in which the quantization error is modelled by an additive noise source.

The additive noise source can be defined as:

$$e_i(n) = R_i(n) - R_i(n-1) \tag{8}$$

where

$$R_i(n) = x_i - Q[x_i] \tag{9}$$

$Q[x_i]$ denotes the quantized value of x_i.

The system of equations describing the actual output sequence in figure 2 is:

$$\left.\begin{aligned}
y_s(n) &= y_s(n-1) + K2^{-b_3}v_2(n-1) - [R_3(n) - R_3(n-1)]\\
v_1(n) &= 2^{-b_1}y_s(n) - [R_1(n) - R_1(n-1)]\\
y_2(b) &= y_2(n-1) - v_1(n)\\
y_3(n) &= \tfrac{1}{2}v_1(n) + y_2(n) - R_4(n)\\
v_2(n) &= 2^{-b_2}y_2(n) - [R_2(n) - R_2(n-1)]\\
y_c &= \frac{K2^{-(b_2+b_3)}}{\sin\theta}y_3(n) - R_5(n)
\end{aligned}\right\} \tag{10}$$

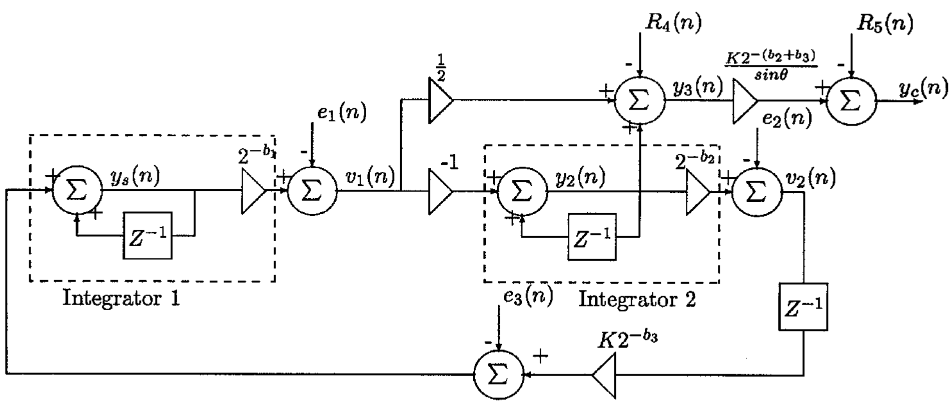

Figure 2. Linearized block diagram model of the complex digital oscillator.

where

$$R_4(n) = \tfrac{1}{2} v_1(n) - Q\left[\frac{v_1(n)}{2}\right] \tag{11}$$

and

$$R_5(n) = \frac{K2^{-(b_2+b_3)}}{\sin\theta} y_3(n) - Q\left[\frac{K2^{-(b_2+b_3)}}{\sin\theta} y_3(n)\right] \tag{12}$$

$R_i(n)$, $i = 1, 2, 3$, can be considered as a random variable uniformly distributed in the interval $[-1/2, 1/2]$. It is obvious that $R_4(n)$ and $R_5(n)$ are not accumulated with time. Therefore, they can be neglected because their contribution to the output error sequence is very small.

The system shown in figure 2 was analysed in Al-Ibrahim and Al-Khateeb (1996 b). It was shown that the output can be expressed in the Z-transform as the sum of a sinusoidal-form term plus a noise term as:

$$Y_c(Z) = W_c(Z) + ER_c(Z) \tag{13}$$

$$Y_s(Z) = W_s(Z) + ER_s(Z) \tag{14}$$

where $W_c(Z)$ and $W_s(Z)$ denote the Z-transform of the oscillator outputs in the absence of roundoff errors. $ER_c(Z)$ and $ER_s(Z)$ are the Z-transform of the output components due to error source.

The signal-to-noise ratio (SNR) of the output $y_c(n)$ and $y_s(n)$ were derived in Al-Ibrahim and Al-Khateeb (1996b). The improvement factor of the complex oscillator is defined as the ratio of the SNR of the jth $(j = 1, 2)$ complex oscillator output and the SNR of the direct digital oscillator (Fliege and Wintermantel 1992). However, it was shown that improvement factors are very large (Al-Ibrahim and Al-Khateeb 1996 b) for the complex digital oscillator, and hence the SNR of the complex oscillator structure is several orders of magnitude larger than that of the direct-form one.

4. The proposed digital oscillator structure

The proposed oscillator structure consists of two complex digital oscillators, arranged as shown in figure 3. Each of the complex oscillators is capable of generating both sine and cosine signals.

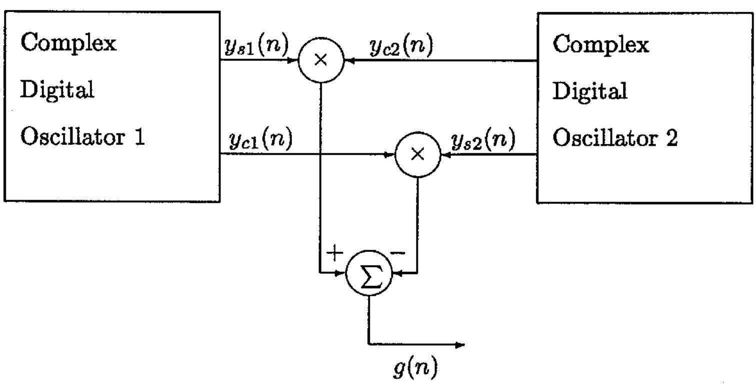

Figure 3. Schematic diagram of the proposed oscillator structure.

Let y_{s1} and y_{c1} be the sine and cosine sequences at the output of the first complex oscillator. Similarly, let y_{s2} and y_{c2} be the sine and cosine sequences at the output of the second complex oscillator. It should be emphasized that each complex oscillator in the proposed structure can generate sinusoidal waveforms separately and independently from the other oscillator. In other words, the amplitude and frequency of the first oscillator output can be chosen independently of their counterparts in the second oscillator.

Assume that the number of bits used for implementing the gains of the integrators in the jth $(j = 1, 2)$ complex digital oscillator is the same $(b_{1j} = b_1, b_{2j} = b_2)$. Also assume that the number of bits used for implementing the multipliers in the jth complex digital oscillator is the same $(b_{3j} = b_3)$. Furthermore, let all the initial conditions be zeros except $y_{21}(-1)$ and $y_{22}(-1)$, where y_{2j} $(j = 1, 2)$ is the value of the initial condition of the jth complex digital oscillator. Then:

$$y_{s1}(n) = \frac{K_1 2^{-(b_2+b_3)}}{\sin\theta_1'} y_{21}(-1)\sin n\theta_1' + e_{s1}(n) \tag{15}$$

$$y_{c1}(n) = \frac{K_1 2^{-(b_2+b_3)}}{\sin\theta_1'} y_{21}(-1)\cos n\theta_1' + e_{c1}(n) \tag{16}$$

$$y_{s2}(n) = \frac{K_2 2^{-(b_2+b_3)}}{\sin\theta_2'} y_{22}(-1)\sin n\theta_2' + e_{s2}(n) \tag{17}$$

$$y_{c2}(n) = \frac{K_2 2^{-(b_2+b_3)}}{\sin\theta_2'} y_{22}(-1)\cos n\theta_2' + e_{c2}(n) \tag{18}$$

Observe that θ_j' $(j = 1, 2)$ denotes the actual angle of the jth pole and is determined by the jth implemented multiplier coefficient, $2\cos\theta_j' = Q[2\cos\theta_j]$. Assuming that $\theta_1' \neq \theta_2'$ and the error sequences are all zeros, the output $g(n)$ in figure 3 is given by

$$g(n) = y_{s1}(n)y_{c2}(n) - y_{c1}(n)y_{s2}(n) \tag{19}$$

Upon simplifying the last equation, it follows that

$$g(n) = \frac{K_1K_2 2^{-2(b_2+b_3)}}{\sin(\theta_1')\sin(\theta_2')} y_{21}(-1)y_{22}(-1)[\sin(n\theta_1')\cos(n\theta_2') - \cos(n\theta_1')\sin(n\theta_2')] \tag{20}$$

Using trigonometric identities, the last equation can be rewritten as:

$$g(n) = \frac{K_1K_2 2^{-2(b_2+b_3)}}{\sin(\theta_1')\sin(\theta_2')} y_{21}(-1)y_{22}(-1)\sin[n(\theta_1' - \theta_2')] \tag{21}$$

Equation (21) shows that the output of our proposed oscillator is a sinusoidal signal. The frequency of the sinusoidal term is given by

$$f_g = \frac{\theta_1' - \theta_2'}{2\pi T} \tag{22}$$

where T is the sampling interval. From (22), it is obvious that $f_g = f_1 - f_2$, and the number of samples per cycle of the generated sinusoid is

$$N_g = \frac{2\pi}{\theta_1' - \theta_2'} = \frac{N_1N_2}{N_2 - N_1} \tag{23}$$

where $N_j = 2\pi/\theta_j'$, $j = 1, 2$.

Therefore, if we choose $\theta_2' = \cos^{-1}(\alpha_2'/2)$, $\theta_1' = \cos^{-1}[(\alpha_2'/2) + (2^{-(b_1+b_2+b_3)}/2)]$, and small values of α_2', the generated frequency will be very much smaller than that of the direct-form digital oscillator (Fliege and Wintermantel 1992). In fact, the values of θ_1' and θ_2' are chosen in such a manner as to have a very small difference $(\theta_1' - \theta_2')$ by choosing $K_2 - K_1 = 1$.

The sensitivity of the complex digital oscillator as given in Hartimo (1983) is a measure of the spacing between adjacent poles. Let the multiplier coefficient α' be increased by $2^{-(b_1+b_2+b_3)}$; it follows that the angle of the pole θ' is increased by:

$$\Delta\theta' \simeq \frac{-2^{-(b_1+b_2+b_3+1)}}{\sin\theta'} \tag{24}$$

In the proposed structure, the frequency is varied by varying θ_1' or θ_2' or both. However, let both α_1' and α_2' be increased by $2^{-(b_1+b_2+b_3)}$; it follows that the angles θ_1' and θ_2' will be incremented by

$$\Delta\theta_1' \simeq \frac{-1}{\sin\theta_1'} 2^{-(b_1+b_2+b_3+1)} \tag{25}$$

and

$$\Delta\theta_2' \simeq \frac{-1}{\sin\theta_2'} 2^{-(b_1+b_2+b_3+1)} \tag{26}$$

respectively. The increment in the difference $\theta_1' - \theta_2'$ is given by

$$\Delta(\theta_1' - \theta_2') \simeq -\left(\frac{1}{\sin\theta_1'} - \frac{1}{\sin\theta_2'}\right) 2^{-(b_1+b_2+b_3+1)} \tag{27}$$

Define η as the ratio of $\Delta(\theta_1' - \theta_2')$ and $\Delta\theta'$, to obtain

$$\eta = \frac{\sin\theta'}{\sin\theta_1'} - \frac{\sin\theta'}{\sin\theta_2'} \tag{28}$$

Let $\theta' = \theta_1'$; i.e. the first oscillator in our proposed system is operating at a frequency equal to that of the direct-form oscillator. Therefore, η reduces to

$$\eta = 1 - \frac{\sin\theta_1'}{\sin\theta_2'} \tag{29}$$

Expressing both θ_1' and θ_2' in terms of the two consecutive multiplier coefficients, we obtain

$$\eta = 1 - \sqrt{\frac{1 - [(\alpha'/2) + 2^{-(b_1+b_2+b_3+1)}]^2}{1 - (\alpha'/2)^2}} \tag{30}$$

Simplifying (30), we obtain

$$\eta \simeq \frac{\alpha' 2^{-(b_1+b_2+b_3+2)}}{1 - (\alpha'/2)^2} \tag{31}$$

Equation (31) shows that η is small for large b's, and decreases by decreasing α' for fixed b's. The values of the coefficients that give the smallest difference between θ_1' and θ_2' are $\alpha_2' = 2(1 - (K_2 2^{-(b_1+b_2+b_3)}/2))$ and $\alpha_1' = \alpha_2' + 2^{-(b_1+b_2+b_3)}$. This occurs when $K_2 - K_1 = 1$. Hence, the smallest value of θ', $\theta_{g\,\min}$ which can be implemented by the proposed structure is given by

$$\theta_{g\,min} = \cos^{-1}\left[\frac{\alpha_2'}{2}\right] - \cos^{-1}\left[\frac{\alpha_2'}{2} + \frac{2^{-(b_1+b_2+b_3)}}{2}\right] \tag{32}$$

The maximum number of samples per cycle can be given by

$$N_{g\,max} = \frac{2\pi}{\theta_{g\,min}} \tag{33}$$

Consequently, the smallest frequency which the oscillator can generate is

$$f_{g\,min} = \frac{\theta_{g\,min}}{2\pi T} \tag{34}$$

Thus, the proposed oscillator generates extremely low frequencies and the generated frequency decreases as the value of K_1 and K_2 increases.

5. Numerical examples and simulation results

To illustrate the performance of the proposed oscillator, let the coefficients of the multiplier of the two complex digital oscillators be chosen as $\alpha_1' = 2\cos\theta_1' = 1.9993744$ and $\alpha_2' = 2\cos\theta_2' = 1.9993668$, with $b_1 = b_2 = 5$ bits, and the multiplier coefficients $b_3 = 7$. Such values for the multiplier coefficients can be achieved with $K_1 = 82$ and $K_2 = 83$. Hence $\theta_1' = 1.4331318^\circ$ and $\theta_2' = 1.441844^\circ$. According to equation (23), $N_1 = 251.19811$, $N_2 = 249.68027$ samples per cycle. From (23), it follows that $N_g = 4.1321 \times 10^4$ samples per cycle, with $\Delta\theta' = (8.1722 \times 10^{-3})^\circ$. Comparing the generated number of samples per cycle using the proposed structure with that using the direct-form digital oscillator, with seven bits used to implement the multiplier coefficients in both cases, the latter case needs 26 bits for the fractional part to produce the same number of samples. Thus, the proposed structure saves 19 bits for the fractional part. Moreover, the maximum number of samples that can be generated using the direct-form oscillator is $N_{max} = 71.063$ when seven bits are used for the fractional part.

On the other hand, the maximum number of samples that can be generated using the efficient low frequency digital sinusoidal oscillator (Al-Ibrahim and Al-Khateeb 1996 a) is $N_{max} = 1608.4648$ samples per cycle when seven bits are used for the fractional part of the multiplier coefficients. However, the proposed structure can generate 1789.1782 samples per cycle with three bits $(b_3 = 3)$ used for implementing the fractional part of the multiplier coefficients, and $b_1 = b_2 = 5$ when $K_1 = 2$ and $K_2 = 3$. Thus, the proposed structure saves four bits in the fractional part.

The proposed digital oscillator was simulated by a MATLAB program, and the results are shown in table 1. The initial conditions, $y_{21}(-1)$ and $y_{22}(-1)$, are chosen to be equal to 104. The table gives the values of K_1, where K_2 is chosen to be equal to $K_1 + 1$. The width of the multiplier coefficients $(K_i 2^{-b_3},\ i = 1,2)$ and the adders is seven bits $(b_3 = 7)$ for the fractional part, one bit for the integer part and one bit for the sign, all in sign and magnitude number representation. The quantization performed is rounding. The proposed oscillator uses five bits $(b_1 = b_2 = 5)$ for implementing the gains of the integrators. The table gives the generated number of samples per cycle, N_g. It is seen from table 1 that the generated number of samples per cycle of the proposed oscillator is extremely large.

K_1	N_g	f_g (Hz)	THD_g (%)
30	2.5124×10^4	597.0404	0.0096
31	2.5533×10^4	587.4865	0.0083
32	2.5935×10^4	578.3771	0.0079
33	2.6331×10^4	569.6787	0.0079
34	2.6721×10^4	561.3614	0.0076
35	2.7105×10^4	553.3982	0.0089
64	3.6535×10^4	410.5599	0.0078
65	3.6818×10^4	407.4141	0.0081
66	3.7097×10^4	404.3395	0.0074
67	3.7375×10^4	401.3395	0.0068
68	3.7651×10^4	398.3927	0.0090
69	3.7925×10^4	395.5174	0.0075
80	4.0816×10^4	367.5056	0.0067
81	4.1068×10^4	365.5086	0.0071
82	4.1319×10^4	363.0242	0.0078
83	4.1569×10^4	360.8442	0.0071
84	4.1817×10^4	358.7050	0.0088
85	4.2064×10^4	356.5994	0.0079

Table 1. Parameters of the proposed oscillator structure and the generated sinusoidal signals for $f_c = 15$ MHz.

Assuming a clock frequency $f_c = 1/T = 15\,\text{MHz}$, table 1 shows that the generated frequencies, f_g, of the proposed structure are extremely low and decrease as the value of K_1 and K_2 increases.

One important characteristic of the digital oscillator is the total harmonic distortion, THD, which is defined as

$$\text{THD}\% = \frac{E_T - E(f_0)}{E_T} \times 100\% \tag{35}$$

where E_T is the total energy of the waveform, $E(f_0)$ is the energy of the fundamental frequency f_0.

The THD% for the generated waveform samples were computed using a MATLAB program. Table 1 shows that the THD% of the generated waveform, THD_g is less than $1 \times 10^{-2}\%$

However, the THD% of the generated signals in the proposed structure is of the same order as that of the extremely low sensitivity oscillator (Al-Ibrahim and Al-Khateeb 1996 b) and several orders of magnitude less than that of the efficient low-frequency digital oscillator (Al-Ibrahim and Al-Khateeb 1996 a) with no filtering needed at the output. The performance of the proposed oscillator structure as reflected by the simulation results is essentially constant over the frequency range presented.

6. Conclusion

An efficient extremely low frequency digital sinusoidal oscillator with high resolution and extremely low sensitivity is presented. The proposed oscillator structure can generate frequencies much lower than all structures known in the literature without the need to increase the width of the multiplier coefficient. The sensitivity to coefficient quantization of the proposed structure is several orders of magnitude less than that of the direct-form digital oscillator. It is shown that the proposed structure has simple implementation because it can be implemented on an already available signal processor. The frequency of the proposed structure covers a wide range with high resolution. Simulation results obtained confirm the analysis and show that a low level of harmonic distortion is achieved for the generated sinusoids.

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